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MEISSNER EFFECT FOR COLOR SUPERCONDUCTING QUARK MATTER

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The behaviour of the magnetic field inside the superconducting quark matter core of a neutron star is investigated in the framework of the Ginzburg–Landau theory. We take into account the simultaneous coupling of the diquark condensate field to the usual magnetic and to the gluomagnetic gauge fields. We solve the problem for three different physical situations: a semi-infinite region with a planar boundary, a spherical region, and a cylindrical region. We show that Meissner currents near the quark core boundary effectively screen the external static magnetic field.

INTRODUCTION

Recently, possible formation of diquark condensates in QCD at finite density has been reinvestigated in a series of papers following Refs. 1, 2. It has been shown that in chiral quark models with nonperturbative four-point interaction motivated from instantons [3] or nonperturbative gluon propagators [4, 5], the anomalous quark pair amplitudes in the color antitriplet channel can be very large: of the order of 100 MeV. Therefore, one expects the diquark condensate to dominate the physics at densities beyond the deconfinement/chiral restoration transition density and below the critical temperature (of the order of 50 MeV). Various phases are possible. The so-called two-flavor (2SC) and three-flavor (3SC) phases allow for unpaired quarks of one color. It has been also found [6,7] that there can exist a color-flavor locked (CFL) phase for not too large strange quark masses [8], where color superconductivity is complete in the sense that diquark condensation results in a pairing gap for the quarks of all three flavors and colors.

The high-density phases of QCD at low temperatures are relevant for the explanation of phenomena in rotating massive compact stars which might manifest themselves as pulsars. Physical properties of these objects (once being measured) could constrain our hypotheses about the state of matter at the extremes of densities. In contrast to the situation for the cooling behaviour of compact stars [9,10], where the CFL phase is dramatically different from the 2SC and 3SC phases, we don't expect qualitative changes of the magnetic field structure for these phases. Therefore, below we shall restrict ourselves to the discussion of the simpler two-flavor theory first. Bailin and Love [11] used a perturbative gluon propagator which yielded a very small pairing gap and they concluded that quark matter is a type I superconductor, which expells the magnetic field of a neutron star within time-scales of 10^4 y. If their arguments would hold in general, the observation of life-times for magnetic fields as large as 10^7 y [12, 13] would exclude the occurrence of an extended superconducting quark matter core in pulsars. These estimates are not valid for the case of diquark condensates characterized by large quark gaps. Besides, in Ref. 14 the authors found that within recent nonperturbative approaches for the effective quark interaction that allow for large pairing gaps, the quark condensate forms a type II superconductor. Consequently for the magnetic field $H < H_{c_1}$ there exists a Meissner effect and for $H_{c_2} > H > H_{c_1}$ the magnetic field can penetrate into the quark core in quantized flux tubes. However, they have not considered in that paper the simultaneous coupling of the quark fields to the magnetic and gluomagnetic gauge fields.

Though color and ordinary electromagnetism are broken in a color superconductor, there is a linear combination of the photon and the gluon that remains massless. The authors of Ref. 15 have considered the problem of the presence of magnetic fields inside color superconducting quark matter taking into account the possibility of the so-called «rotated electromagnetism». They came to the conclusion that there is no Meissner effect and the external static homogeneous magnetic field can penetrate into superconducting quark matter because in their case it obeys the sourceless Maxwell equations. To our opinion this result is obtained when one does not pose correct boundary conditions for the fields. Obviously it is energetically favorable to expell the magnetic field rather than to allow its penetration inside the superconducting matter. Using for the description of the diquark condensate, interacting with two gauge fields, the same model as in Refs. 9, 8, 16, we will show that the presence of the massless excitation in the spectrum does not prevent the Meissner currents to effectively screen the static external magnetic field.

In Ref. 16 two of us have derived the Ginzburg–Landau equations of motion for the diquark condensate placed in static magnetic and gluomagnetic fields,

$$ad_p + \beta(d_p d_p^*)d_p + \gamma \left(i \nabla - \frac{e}{3}\mathbf{A} + \frac{g}{\sqrt{3}}\mathbf{G}_8 \right)^2 d_p = 0, \tag{1}$$

where d_p is the order parameter; $a = t\ dn/dE$; $\beta = (dn/dE)\ 7\zeta(3)(\pi T_c)^{-2}/8$;

$\gamma = p_F^2\ \beta/(6\mu^2)$; $dn/dE = p_F\ \mu/\pi^2$; $t = (T - T_c)/T_c$; T_c being the critical temperature, p_F — the quark Fermi momentum, and for the gauge fields

$$\lambda_q^2 \,\text{rot rot}\, \mathbf{A} + \sin^2\alpha\, \mathbf{A} = i\frac{\sin\,\alpha(d_p \nabla d_p^* - d_p^* \nabla d_p)}{2qd^2} + \sin\,\alpha\,\cos\,\alpha\, \mathbf{G}_8, \quad (2)$$

$$\lambda_q^2 \,\text{rot rot}\, \mathbf{G}_8 + \cos^2\alpha\, \mathbf{G}_8 = -i\frac{\cos\,\alpha(d_p \nabla d_p^* - d_p^* \nabla d_p)}{2qd^2} + \sin\,\alpha\,\cos\,\alpha\, \mathbf{A}. \quad (3)$$

These equations introduce a «new» charge of the diquark pair $q = \sqrt{\eta^2 e^2 + g^2} P_8$, $P_8 = 1/\sqrt{3}$, and for the diquark condensate with paired blue-green and green-blue ud quarks one has $\eta = 1/\sqrt{3}$. The penetration depth of the magnetic and gluomagnetic fields λ_q and the mixing angle α are given by

$$\lambda_q^{-1} = qd\sqrt{2\gamma}, \;\; \cos\alpha = \frac{g}{\sqrt{\eta^2 e^2 + g^2}}. \quad (4)$$

At neutron star densities gluons are strongly coupled ($g^2/4\pi \sim 1$) whereas photons are weakly coupled ($e^2/4\pi = 1/137$), so that $\alpha \simeq \eta e/g$ is small. For $g^2/4\pi \simeq 1$ we get $\alpha \simeq 1/20$. The new charge q is by an order of magnitude larger than $e/\sqrt{3}$.

Please notice also that since red quarks are normal in the 2SC and 3SC phases, there exist the corresponding normal currents $j_\mu^r(A) = -\Pi_{\mu\nu}^{el} A^\nu$ and $j_\mu^r(G_8) = -\Pi_{\mu\nu}^{gl} G_8^\nu$ which, however, do not contribute in the static limit under consideration to the above Ginzburg–Landau equations, cf. [17]. Thus, the qualitative behavior of the static magnetic field for all three 2SC, 3SC, and CFL phases is the same.

We will solve the Ginzburg–Landau equations (1)–(3) for the case of a homogeneous external magnetic field for three types of superconducting regions: a) a semi-infinite region with a planar boundary, b) a cylindrical region and c) a spherical region. The cases a) and c) simulate the behavior of the magnetic field in quark cores of massive neutron stars. A discussion of all the three cases might be interesting in connection with the expectation that the slab, the rod and the droplet structures may exist within the mixed quark-hadron phase of the neutron stars, cf. [18, 19].

We assume a sharp boundary between the quark and hadron matter since the diffusion boundary layer is thin, of the order of the size of the confinement radius ($\sim 0.2 \div 0.4$ fm), and we suppose that the coherence length $l_\xi = \sqrt{\gamma/(-2a)}$ is not less than this value and the magnetic and gluomagnetic field penetration depth λ_q is somewhat larger than the confinement radius. Also we assume that the size of the quark region is much larger than all mentioned lengths.

SOLUTION OF GINZBURG–LANDAU EQUATIONS

Let us rewrite equations (2) and (3) for a homogeneous superconducting matter region (being either a type I superconductor, or a type II superconductor for $H < H_{c_1}$) in the following form

$$\lambda_q^2 \,\mathrm{rot}\,\mathrm{rot}\,\mathbf{A} + \sin^2\alpha\,\mathbf{A} = \sin\alpha\,\cos\alpha\,\mathbf{G}_8, \tag{5}$$

$$\lambda_q^2 \,\mathrm{rot}\,\mathrm{rot}\,\mathbf{G}_8 + \cos^2\alpha\,\mathbf{G}_8 = \sin\alpha\,\cos\alpha\,\mathbf{A}. \tag{6}$$

The field $\mathbf{G}_8$ is defined from (5) as follows

$$\mathbf{G}_8 = \frac{\lambda_q^2 \,\mathrm{rot}\,\mathrm{rot}\,\mathbf{A} + \sin^2\alpha\,\mathbf{A}}{\sin\alpha\,\cos\alpha}. \tag{7}$$

From equations (6) and (7) we obtain the relation

$$\mathrm{rot}\,\mathrm{rot}\,\mathbf{G}_8 = -\cot\alpha\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{A}. \tag{8}$$

Substitution of $\mathbf{G}_8$ from (7) into (8) yields

$$\mathrm{rot}\,\mathrm{rot}\,(\lambda_q^2 \,\mathrm{rot}\,\mathrm{rot}\,\mathbf{A} + \mathbf{A}) = 0. \tag{9}$$

Introducing the new function $\mathbf{M}$,

$$\mathbf{M} = \mathrm{rot}\,\mathrm{rot}\,\mathbf{A}, \tag{10}$$

we obtain

$$\lambda_q^2 \,\mathrm{rot}\,\mathrm{rot}\,\mathbf{M} + \mathbf{M} = 0. \tag{11}$$

Thus the vector potential $\mathbf{A}$ can be determined by simultaneous solution of equations (10) and (11), whereas the gluonic potential $\mathbf{G}_8$ is found from (7).

For the solution of equations (7), (10), and (11) we also need appropriate boundary conditions. At the quark-hadronic matter boundary we require the continuity of the magnetic field and the vanishing of the gluon potential ($\mathbf{G}_8 = 0$) due to gluon confinement. Also the potential $\mathbf{G}_8$ and the magnetic induction cannot be infinite within the region of their existence. As we shall see below, these conditions are sufficient for a unique determination of the magnetic and gluomagnetic fields inside the quark matter.

Equations (1), (2), and (3) have an obvious solution for a homogeneous ($\nabla d_p = 0$) and infinite superconductor ($\mathbf{A} = 0$, $\mathbf{G}_8 = 0$)

$$d^2 = -a/\beta > 0. \tag{12}$$

This solution motivates the possibility of existence of the complete Meissner effect for both the fields $\mathbf{A}$ and $\mathbf{G}_8$ inside the quark superconductor. It corresponds to the absolute minimum value of the free energy $f = f_n - a^2/(2\beta)$, where f_n is the free energy of the normal quark matter [11, 14]. The presence of the fields $\mathbf{A} \neq 0$, $\mathbf{G}_8 \neq 0$ inside the quark region would increase the free energy.

As we have mentioned, the estimates [14, 16] have demonstrated that color superconductors are type II superconductors. Indeed, the Ginzburg–Landau parameter $\kappa = \lambda_q/l_\xi = \sqrt{\beta}/(\gamma q) > 3$. For type II superconductors one can drop the fields $\mathbf{A}$ and $\mathbf{G}_8$ in the Ginzburg–Landau equation (1) arriving at the solution (12), since the coherence length l_ξ is smaller than the penetration depth of the magnetic and gluomagnetic fields λ_q. Then in equations of motion (2) and (3) the penetration depth of the magnetic and gluomagnetic fields λ_q can be put constant. This simplifies solution of the Ginzburg–Landau equations very much. For further analytical treatment of the problem we will assume that we deal with a type II superconductor although our main conclusion on the existence of the Meissner effect is quite general.

Planar Boundary. Let us first consider a semi-infinite region of superconducting quark matter for $x < 0$ with a planar boundary, which coincides with the zy plane. The external static homogeneous magnetic field H is directed along the z axis, the external vector potential $\mathbf{A}$ is aligned in the y direction $A_y(x) = Hx$. Then the internal potentials $\mathbf{A}$ and $\mathbf{G}_8$ also have only y components: $A_y = A_y(x)$ and $G_{8y} = G_{8y}(x)$, $\operatorname{div}\mathbf{A} = 0$, $\operatorname{div}\mathbf{G}_8 = 0$. Simultaneous solution of equations (10) and (11) determines the electromagnetic vector potential A_y as follows

$$A_y = c_1 \exp\left(\frac{x}{\lambda_q}\right) + c_2 x + c_3. \tag{13}$$

We put $c_2 = 0$ because otherwise $A_y \to \infty$ and $G_{8y} \to \infty$ for $x \to \infty$ that would lead to a complete destruction of the condensate. We search for an energetically favorable unique solution of the problem satisfying the above-mentioned boundary conditions. Thus we further put

$$A_y = c_1 \exp\left(\frac{x}{\lambda_q}\right) + c_3. \tag{14}$$

Substitution of the solution (14) into equation (7) yields the gluonic potential G_{8y},

$$G_{8y} = -\cot\alpha\, c_1 \exp\left(\frac{x}{\lambda q}\right) + \tan\alpha\, c_3. \tag{15}$$

The boundary condition $G_{8y}(x=0)=0$ determines the constant $c_3 = c_1 \cot^2 \alpha$. For the vector potential of the magnetic field we obtain

$$A_y = c_1 \left(\exp\left(\frac{x}{\lambda_q}\right) + \cot^2 \alpha \right). \tag{16}$$

We can determine c_1 from the remaining boundary condition $dA_y(x=0)/dx = 0$ which yields $c_1 = H\lambda_q$. Finally the potentials render

$$A_y = H\lambda_q \left(\exp\left(\frac{x}{\lambda_q}\right) + \cot^2 \alpha \right), \tag{17}$$

$$G_{8y} = -\cot\,\alpha\, H\,\lambda_q \left(\exp\left(\frac{x}{\lambda_q}\right) - 1 \right). \tag{18}$$

Then, for the magnetic induction $B_z = dA_y/dx$ and for the gluonic field $K_z = dG_{8y}/dx$ inside the quark superconductor, we have the following expressions

$$\mathbf{B} = H \exp\left(\frac{x}{\lambda_q}\right) \mathbf{e}_z, \tag{19}$$

$$\mathbf{K} = -\cot\alpha\, B\, \mathbf{e}_z. \tag{20}$$

Ref. 15 introduced the «rotated» fields $\mathbf{B}^x$ and $\mathbf{B}^y$,

$$\mathbf{B}^x = -\sin\alpha\,\mathbf{B} + \cos\,\alpha\,\mathbf{K}, \tag{21}$$

$$\mathbf{B}^y = \cos\,\alpha\,\mathbf{B} + \sin\,\alpha\,\mathbf{K}. \tag{22}$$

It is easy to see that solutions (19) and (20) yield $B_z^y = 0$ and $B_z^x = -(H/\sin\,\alpha)\exp\left(x/\lambda_q\right)$. Therefore there is no $\mathbf{B}^y$ field inside the superconducting quark matter and the B_z^x field is expelled from the color superconductor. Consequently, in contradiction with the statement of Ref. 15 there is a Meissner effect for the color superconductors.

Cylindrical Structures. Now we shall consider a cylindrical region of superconducting quark matter of radius a, whose axis coincides with z axis of cylindrical coordinates. Such a situation may occur in the mixed phase where the rods of quark matter imbedded in the hadron matter are possible configurations. The external homogeneous magnetic field H is directed along the z axis, the external vector potential A is aligned in the φ direction $A_\varphi = Hr/2$. The internal vector potentials $\mathbf{A}$ and $\mathbf{G}_8$ have only φ components $A_\varphi(r)$ and $G_{8\varphi}(r)$. Thus, equation (11) acquires the form

$$\frac{d^2 M_\varphi}{dr^2} + \frac{1}{r}\frac{dM_\varphi}{dr} - \left(\frac{1}{r^2} + \frac{1}{\lambda^2}\right) M_\varphi = 0. \tag{23}$$

The solution of this equation is $M_\varphi = -c_1 I_1(r/\lambda_q)$, where $I_1(r/\lambda_q)$ is the corresponding modified Bessel function. Consequently, equation (10) can be written in the following form

$$\frac{d^2 A_\varphi}{dr^2} + \frac{1}{r}\frac{dA_\varphi}{dr} - \frac{A_\varphi}{r^2} = c_1 I_1\left(\frac{r}{\lambda_q}\right). \tag{24}$$

For the vector potential A_φ we obtain

$$A_\varphi(r) = c_1 \lambda_q^2 I_1\left(\frac{r}{\lambda_q}\right) + c_2 r. \tag{25}$$

Then for the potential $G_{8\varphi}$ from (7) we find the following expression

$$G_{8\varphi}(r) = -\cot\alpha\, c_1\, \lambda_q^2 I_1\left(\frac{r}{\lambda_q}\right) + \tan\alpha\, c_2 r. \tag{26}$$

The magnetic induction B_z inside the superconductor is given by

$$B_z(r) = c_1 \lambda_q I_0\left(\frac{r}{\lambda_q}\right) + 2c_2. \tag{27}$$

We determine c_2 from the boundary condition $G_{8\varphi}(a) = 0$ and c_1 from $B_z(a) = H$. Consequently, final expressions for the potentials are

$$A_\varphi(r) = \frac{H\lambda_q}{P(a/\lambda_q,\alpha)}\left[I_1\left(\frac{r}{\lambda_q}\right) + \cot^2\alpha\,\frac{r}{a} I_1\left(\frac{a}{\lambda_q}\right)\right], \tag{28}$$

$$G_{8\varphi}(r) = -\frac{H\lambda_q\cot\alpha}{P(a/\lambda_q,\alpha)}\left[I_1\left(\frac{r}{\lambda_q}\right) - \frac{r}{a} I_1(\frac{a}{\lambda_q})\right], \tag{29}$$

and for the corresponding fields we get

$$B_z(r) = \frac{H}{P(a/\lambda_q,\alpha)}\left[I_0\left(\frac{r}{\lambda_q}\right) + 2\cot^2\alpha\,\frac{\lambda_q}{a} I_1\left(\frac{a}{\lambda_q}\right)\right], \tag{30}$$

$$K_z(r) = -\frac{H\cot\alpha}{P(a/\lambda_q,\alpha)}\left[I_0\left(\frac{r}{\lambda_q}\right) - 2\frac{\lambda_q}{a} I_1\left(\frac{a}{\lambda_q}\right)\right], \tag{31}$$

where $P(a/\lambda_q, \alpha)$ is given by

$$P(a/\lambda_q, \alpha) = I_0\left(\frac{a}{\lambda_q}\right) + 2\frac{\lambda_q}{a}\cot^2\alpha\, I_1\left(\frac{a}{\lambda_q}\right). \tag{32}$$

Thus, we obtain the following final expressions for the rotated fields

$$B_z^y = \frac{2\lambda_q H \cot\alpha}{aP(a/\lambda_q, \alpha)\sin\alpha} I_1\left(\frac{a}{\lambda_q}\right), \tag{33}$$

$$B_z^x = -\frac{H}{P(a/\lambda_q)\sin\alpha} I_0\left(\frac{r}{\lambda_q}\right). \tag{34}$$

We notice that the field $\mathbf{B}^y$ is homogeneous inside the quark matter. Thus one may expect that in presence of the magnetic field the slab structures are energetically favorable starting with a smaller quark fraction volume than in the absence of the magnetic field since, as we have argued, the magnetic field is expelled from the slabs and it penetrates the cylinders. In application to the description of the quark core of the neutron star, the discussion of the cylindrical case has no meaning and one should further consider the case of a spherical geometry.

Spherical Case. We now assume that the neutron star possesses a spherical core of radius a consisting of color superconducting quark matter. The applied homogeneous magnetic field H is directed along the z axis. The functions $\mathbf{M}$, $\mathbf{A}$ and $\mathbf{G}_8$ have only φ components: $M_\varphi(r,\vartheta)$, $A_\varphi(r,\vartheta)$ and $G_{8\varphi}(r,\vartheta)$. For the solution of the equation (11) we use the ansatz $M_\varphi(r,\vartheta) = f(r)\sin\vartheta$. Then equation (11) can be written in spherical coordinates as

$$\frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr} - \left(\frac{2}{r^2} + \frac{1}{\lambda_q^2}\right) f = 0. \tag{35}$$

The solution of equation (35) which tends to zero at the centre of the core renders

$$f(r) = -\frac{D}{r^2} J\left(\frac{r}{\lambda_q}\right), \tag{36}$$

where

$$J(\frac{r}{\lambda_q}) = \sinh\left(\frac{r}{\lambda_q}\right) - \frac{r}{\lambda_q}\cosh\left(\frac{r}{\lambda_q}\right). \tag{37}$$

For the solution of equation (10) we use the ansatz $A_\varphi(r,\vartheta) = g(r)\sin\vartheta$. Then equation (10) acquires the form

$$\frac{d^2 g}{dr^2} + \frac{2}{r}\frac{dg}{dr} - \frac{2g}{r^2} = \frac{D}{r^2} J\left(\frac{r}{\lambda_q}\right). \tag{38}$$

The general solution of equation (10) in spherical coordinates is given by

$$A_\varphi(r,\vartheta) = \frac{D\lambda_q^2}{r^2} J\left(\frac{r}{\lambda_q}\right)\sin\vartheta + c_1 r\sin\vartheta. \tag{39}$$

We also find the general solution for the gluonic potential from (7)

$$G_{8\varphi}(r,\vartheta) = -\cot\alpha\frac{D\lambda_q^2}{r^2} J\left(\frac{r}{\lambda_q}\right)\sin\vartheta + \tan\alpha\, c_1 r\sin\vartheta. \tag{40}$$

The constant c_1 is determined from the boundary condition $G_{8\varphi}(a,\vartheta) = 0$ as $c_1 = D\lambda_q^2\cot^2\alpha J(a/\lambda_q)/a^3$. Then expressions for the vector potential A_φ and for the gluonic potential $G_{8\varphi}$ are written as

$$\begin{aligned} A_\varphi(r,\vartheta) &= \frac{D\lambda_q^2}{r^2}\left[J\left(\frac{r}{\lambda_q}\right) + \cot^2\alpha\frac{r^3}{a^3}J\left(\frac{a}{\lambda_q}\right)\right]\sin\vartheta, \\ G_{8\varphi}(r,\vartheta) &= -\cot\alpha\frac{D\lambda_q^2}{r^2}\left[J\left(\frac{r}{\lambda_q}\right) - \frac{r^3}{a^3}J\left(\frac{a}{\lambda_q}\right)\right]\sin\vartheta. \end{aligned} \tag{41}$$

Therefore, the expressions for the radial component B_{ir} and the tangential component $B_{i\vartheta}$ acquire the form

$$\begin{aligned} B_{ir}(r,\vartheta) &= \frac{2D\lambda_q^2}{a^3}\left[\frac{a^3}{r^3}J\left(\frac{r}{\lambda_q}\right) + \cot^2\alpha\, J\left(\frac{a}{\lambda_q}\right)\right]\cos\vartheta, \\ B_{i\vartheta}(r,\vartheta) &= \frac{D\lambda_q^2}{a^3}\left[\frac{a^3}{r^3}J_1\left(\frac{r}{\lambda_q}\right) - 2\cot^2\alpha\, J\left(\frac{a}{\lambda_q}\right)\right]\sin\vartheta, \end{aligned} \tag{42}$$

where

$$J_1\left(\frac{r}{\lambda_q}\right) = \left(1 + \frac{r^2}{\lambda_q^2}\right)\sinh\left(\frac{r}{\lambda_q}\right) - \frac{r}{\lambda_q}\cosh\left(\frac{r}{\lambda_q}\right). \tag{43}$$

Accordingly, the expressions for the radial component K_{ir} and the tangential component $K_{i\vartheta}$ of the internal gluomagnetic field render

$$\begin{aligned} K_{ir}(r,\vartheta) &= -\cot\alpha\frac{2D\lambda_q^2}{a^3}\left[\frac{a^3}{r^3}\left(\frac{r}{\lambda_q}\right) - J\left(\frac{a}{\lambda_q}\right)\right]\cos\vartheta, \\ K_{i\vartheta}(r,\vartheta) &= -\cot\alpha\frac{D\lambda_q^2}{a^3}\left[\frac{a^3}{r^3}J_1\left(\frac{r}{\lambda_q}\right) + 2J\left(\frac{a}{\lambda_q}\right)\right]\sin\vartheta. \end{aligned} \tag{44}$$

The solutions of the Maxwell equations outside the sphere are

$$B_{er}(r,\vartheta) = \left(H + \frac{2m}{r^3}\right)\cos\vartheta, \quad B_{e\vartheta}(r,\vartheta) = \left(-H + \frac{m}{r^3}\right)\sin\vartheta. \tag{45}$$

We can find D and m from the boundary conditions $B_{ir}(a,\vartheta) = B_{er}(a,\vartheta)$, $B_{i\vartheta}(a,\vartheta) = B_{e\vartheta}(a,\vartheta)$. Thus, we obtain

$$D = -\frac{3Ha}{2N\sinh{(a/\lambda_q)}}, \quad m = -\frac{Ha^3}{2N}\left[1 + 3\frac{\lambda_q^2}{a^2} - 3\frac{\lambda_q}{a}\coth\frac{a}{\lambda_q}\right], \tag{46}$$

where N is given by

$$N = 1 - 3\cot^2\alpha\left[\frac{\lambda_q^2}{a^2} - \frac{\lambda_q}{a}\coth\frac{a}{\lambda_q}\right]. \tag{47}$$

The final expressions for the internal magnetic fields have the form

$$\begin{aligned} B_{ir} &= -\frac{3H\lambda_q^2}{a^2N\sinh{(a/\lambda_q)}}\left[\frac{a^3}{r^3}J\left(\frac{r}{\lambda_q}\right) + \cot^2\alpha J\left(\frac{a}{\lambda_q}\right)\right]\cos\vartheta, \\ B_{i\vartheta} &= -\frac{3H\lambda_q^2}{2a^2N\sinh{(a/\lambda_q)}}\left[\frac{a^3}{r^3}J_1\left(\frac{r}{\lambda_q}\right) - 2\cot^2\alpha J\left(\frac{a}{\lambda_q}\right)\right]\sin\vartheta. \end{aligned} \tag{48}$$

The corresponding internal components of gluomagnetic field are given by

$$\begin{aligned} K_{ir} &= \frac{3H\lambda_q^2\cot\alpha}{a^2N\sinh{(a/\lambda_q)}}\left[\frac{a^3}{r^3}J\left(\frac{r}{\lambda_q}\right) - J\left(\frac{a}{\lambda_q}\right)\right]\cos\vartheta, \\ K_{i\vartheta} &= \frac{3H\lambda_q^2\cot\,\alpha}{2a^2N\sinh{(a/\lambda_{q)}}}\left[\frac{a^3}{r^3}J_1\left(\frac{r}{\lambda_q}\right) + 2J\left(\frac{a}{\lambda_q}\right)\right]\sin\vartheta. \end{aligned} \tag{49}$$

We obtain therefore the following internal components of the rotated sourceless field $\mathbf{B}^y$

$$\begin{aligned} B_{ir}^y &= \frac{3H\lambda_q^2\cot\alpha}{Na^2\sin\,\alpha}\left[\frac{a}{\lambda_q}\coth\frac{a}{\lambda_q} - 1\right]\cos\vartheta, \\ B_{i\vartheta}^y &= -\frac{3H\lambda_q^2\cot\alpha}{Na^2\sin\,\alpha}\left[\frac{a}{\lambda_q}\coth\frac{a}{\lambda_q} - 1\right]\sin\vartheta. \end{aligned} \tag{50}$$

It is to be noticed that the components of the rotated field $\mathbf{B}^y$ depend only on the polar coordinate ϑ. The internal components of the rotated massive field $\mathbf{B}^x$ are

$$\begin{aligned} B_{ir}^x &= \frac{3H\lambda_q^2a}{Nr^3\sin\,\alpha\sinh{(a/\lambda_q)}}J\left(\frac{r}{\lambda_q}\right)\cos\vartheta, \\ B_{i\vartheta}^x &= \frac{3H\lambda_q^2a}{2Nr^3\sin\,\alpha\sinh{(a/\lambda q)}}J_1\left(\frac{r}{\lambda_q}\right)\sin\vartheta. \end{aligned} \tag{51}$$

In a neutron star, the radius of the superconducting quark core a is much larger than the penetration depth λ_q. In this limit the unbroken rotated field components take the form

$$B_{ir}^y = \frac{3H \cot \alpha}{\sin \alpha} \frac{\lambda_q}{a} \cos \vartheta, \quad B_{i\vartheta}^y = -\frac{3H \cot \alpha}{\sin \alpha} \frac{\lambda_q}{a} \sin \vartheta. \tag{52}$$

In the very same limit, for the components of the rotated massive field $\mathbf{B}^x$ at the surface of the quark core we obtain

$$B_{ir}^x(a, \vartheta) = -\frac{3H}{\sin \alpha} \frac{\lambda_q}{a} \cos \vartheta, \quad B_{i\vartheta}^x(a, \vartheta) = \frac{3H}{\sin \alpha} \sin \vartheta. \tag{53}$$

Therefore, the rotated massive field component at the surface of the quark core $B_{i\vartheta}^x(a, \vartheta)$ is larger than $B_{i\vartheta}^y$ by the factor $a/\lambda_q \cot \alpha$, and the $B_{ir}^x(a, \vartheta)$ and B_{ir}^y components are of the same order of magnitude. We notice that there is Meissner effect for the rotated field B^x. Let us now estimate the rotated field $\mathbf{B}^y$ inside the star. For values of the external field strength $H = 10^{12}$ G, the penetration depth $\lambda_q = 1.7$ fm, the mixing angle $\alpha = 0.05$, and the radius of the quark core $a = 1$ km, we obtain $B^y = 2 \cdot 10^{-4}$ G. Thus, we can safely neglect the rotated unbroken field $\mathbf{B}^y$. Therefore we again insist that there is a Meissner effect, and the applied external static magnetic field is thereby almost completely screened within the spherical geometry. Only a tiny fraction of the field can penetrate into the superconducting quark cores of the neutron stars.

CONCLUSION

We have investigated the behaviour of color superconducting quark matter in an external static homogeneous magnetic field and could show that color and electric Meissner currents exist. For this purpose we have solved the Ginzburg–Landau equations for three types of superconducting regions: a) a semi-infinite region with planar boundary, b) a cylindrical region and c) a spherical region. We have obtained analytic expressions for magnetic and gluomagnetic fields inside the quark matter for all three cases. In application to the quark cores of massive neutron stars we showed that one can neglect the rotated field $\mathbf{B}^y$ inside the superconducting quark core since the Meissner currents effectively screen the applied external static magnetic field. These results confirm the physical situation discussed in [16].

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